MULTI-ISSUE BARGAINING UNDER BUDGET CONSTRAINTS

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Daniel Cardona Coll

A B S T R A C T

We analyze a multi-issue bargaining model where the joint production of public goods is budget-constrained. The players must decide the part of the budget that is dedicated to produce any public good. We model the decision process as an alternating o¤er bargaining game with random proposers. The utilities of the players are assumed separable in any public issue. We show that multiple sophisticated outcomes are (generically) attained when a complete agreement is required for a subset of public projects to be implemented, either if the players bargain globally over the sizes of di¤erent public goods or sequentially through partial agreements. However, when public projects are immediately implemented after partial agreements, then uniqueness (which is a necessary condition for e¢ciency) is generically achieved.

Keywords: Multilateral bargaining, Sophisticated equilibrium, Issue-by-issue bargaining, Voting by Quota.

JEL Classi…cation: C72, C78, D72.

1 Introduction

The literature on multi-issue bargaining usually considers situations where the players negotiate over di¤erent goods of …xed size. These studies focus on comparing alternative procedures governing the bargaining and how the speci…c extensive form a¤ects the outcomes. Two main results emerge from this literature. First, if the bargaining proceeds by a sequence of bargaining games with players negotiating separately on each issue then the agenda may play an important role in determining the …nal outcome. The e¤ects of the agenda vanishes as players become patient when consumption occurs only after a complete agreement is reached (see Fershtman (1990)) . However, its e¤ects persist if partial agreements are immediately implemented (Inderst (2000)). Second, we learn that to achieve e¢ciency, the most important issues must be put …rst in the agenda (see Winter (1997)) .

In these papers, the relationship among sequential negotiations is established through the impatience of the players. If the players bargain sequentially (and separately) over issues of …xed size, a player will be willing to accept a small share of some issue when she expects a high utility from the bargaining game on next issue. The reason is that bargaining on next issue occurs only when there is agreement on the actual negotiations. Thus, in such cases, this player behaves as if she is more impatient. This may yield ine¢cient outcomes because of the impossibility of the players to make concessions across issues so that the players may prefer to bargain globally than through a sequence of bargaining games. When the players cannot consume their contingent agreed shares until a complete agreement is reached the link between two (sequential) bargaining games di¤ers from the previous case. Now, players who have assigned a higher contingent utility behave as if they were more impatient and are willing to concede a higher share in actual negotiations to their opponents.

In this paper, we analyze a multi-issue bargaining model where another link among di¤erent bargaining games appears. Consider a set of players endowed with some …xed budget, which can be used to produce some public goods. Once the players decide to produce some amount of a public good, then it is not excludable and there is no rivalry in consumption. Con‡icting interests may appear when deciding the part of the budget assigned to the production of each issue. Since the budget is …xed, the production of a public good of a given size will limit the feasible sizes of other possible public goods. Hence, the bargaining consists of deciding the priorities of the public policy. I.e., the part of the budget which is assigned to each public issue.

In contrast with the traditional analysis of the bargaining problem, two main features must be remarked. First, the set of feasible agreements is not necessarily any allocation of the surplus. The structure of the problem we analyze imposes some restrictions on the set of feasible payo¤ con…gurations. Second, the preferences of the players may be correlated, in the sense that some players may have the same preferences over all possible priorities of the public policy. Fershtman (2000) considers a model where the players may have correlated priorities. He considers a multi-issue bargaining model where the sizes of the goods are …xed and agreement requires only approval of two players. He shows that if the priorities of some players are negatively correlated, then the other player can exploit this in order to achieve a better settlement. This result bears on the fact that meetings are bilateral.

In our model, issues are related through the budget constraint: if the set of players decide to consume a high amount of some public good, then this will limit the consumption on another public good. Thus, if the bargaining proceeds by sequentially deciding the size of any public issue, a clear link is established between actual decisions a future possibilities.

We model the decision process as an alternating o¤er bargaining game with random proposers: at each period, a player is (randomly) chosen as the proposer. Once she makes a proposal, then the rest of players simultaneously reject or accept it. If the number of acceptances exceeds some speci…ed quota q, then this proposal succeeds. Otherwise, a new proposer is chosen. And so on. This model is quite similar to the one where the players have outside options which depend on their valuations of the public issues (see Binmore (1987)) if just two players are involved in the bargaining. For three or more players, multiple sophisticated equilibria are sustained whenever the players cannot be "polarized" into two groups according to their preferences. In such cases, punishment threats can be sustained as part of the equilibrium strategies in an in…nite horizon bargaining game, and this causes multiplicity.

As it is usual in multi-issue bargaining, we analyze two di¤erent scenarios in which the players must decide the priorities of the public policy. A …rst environment where the players propose a complete vector of priorities of the public policy; and a second one, where the bargaining proceeds through a sequence of partial agreements over these priorities. These possibilities induce to di¤erent equilibrium outcomes. Thus, the selection between global bargaining and issue-by-issue bargaining is not immune to strategic manipulation.[1] Moreover, the basic di¤erences between the models we consider are not directly related to the disjunctive between global and/or partial bargaining. They depend on the period in which the public projects are implemented. This distinction is meaningful when players bargain globally over the priorities of the public policy but it is very important when the players bargain issue-by-issue.

In the …rst scenario, the proposals consist of vectors of priorities, which determine the part of the budget dedicated to the production of each public good. This would introduce intransitiveness in majority comparisons when the set of available public projects is greater than two. That is, the set of players cannot generically be polarized into two groups according to their preferences over di¤erent outcomes. This opens the door to reputation possibilities and multiple equilibria are generically sustained. Only when the number of relevant public issues is two then majorities are "stable" and a unique vector of expected weights is attained in equilibrium.

In a second scenario, we develop the idea of issue-by-issue bargaining (see Winter (1997)). The proposals consist of the part of the budget dedicated to produce a particular public good. That is, the players must agree sequentially over the sizes of di¤erent public goods according to some …xed agenda, which determines the order in which di¤erent issues are considered. Using a backwards induction argument one might think that in this framework, the players face to a sequence of bargaining games

[1] Although these strategic incentives are somehow di¤erent from the well-known problem of agenda manipulation, one can found examples where such type of incentives are present. Recently, for instance, some EC countries tried that …sh tari¤s were negotiated separately from other agricultural tari¤s in the recent World Trade Organization meeting in Seattle (December, 1999). Finally, these pressures had no e¤ects and Brussels decided to include …shing tari¤s as part of the global agreement.

where just two issues are considered, and so uniqueness would appear. This is generically true when partial agreements are not contingent to a complete agreement. This unique equilibrium is shown to be dependent on the order in which different weights are submitted to approval. Therefore, the agenda can play an important role in determining the outcome. On the other hand, if a complete agreement is required for partial agreements to be effective, then previous agreements affect future equilibrium outcomes (in some sense, the bargaining becomes global). This induces to a non-bipolar structure of preferences in each subgame, leading to multiplicity.

In the next section, we present the alternating offer bargaining model where proposals consist of vectors of priorities. We analyze the special case where just two public projects are available. The uniqueness result we get in this case is very helpful in section 3 where the bargaining is modeled as a sequence of partial agreements: we show that when partial agreements are not contingent to a complete agreement then uniqueness is generically achieved. In section 4, we discuss the importance of uniqueness in our setting, as a necessary property for efficiency. Section 5 concludes.

2 The Global Agreement Bargaining Framework

There is a finite set of agents $I = \{1, ..., n\}$ who must decide how to distribute a fixed budget of 1 among k different issues. Let K denote this set of issues.[2] Each agent i is defined by a vector $a_i = (a_i(1), ..., a_i(k)) \in \Re^k_+$, where $a_i(j)$ denotes the i's marginal utility of issue j.[3] That is, we assume that the players have additively separable utility functions.

The bargaining among the players is a version of the alternating offer discrete time and infinite horizon bargaining game (see Rubinstein [12] and Binmore [2]). At each period one player is randomly chosen from I as the proposer, with equal probability each. Then, she makes a proposal $x \in \Delta^{k-1} = \left\{(x_1, ..., x_k) \in \Re^k : y_i \geq 0 \text{ for any } i \in K \text{ and } \sum_{i=1}^{k} x_i = 1\right\}$, and the rest of players simultaneously accept or reject it. If at least q players accept x then this proposal succeeds and the budget is assigned according to this vector of priorities. Otherwise, x is rejected and the game moves to next period where a new proposer is randomly chosen.[4] We consider that the players observe only the outcome of the voting stage so that their actions are not observed.

As we noticed in the introduction, a particular environment could be the one where a set of players decides how to invest in different public projects. There is a fixed budget that can be used to produce (continuous) public goods. The players must decide how to spend on different issues. Hence, they are implicitly deciding how to consume of each public good. These public goods are not excludable and there is no rivalry in consumption once they are produced. However, conflicting interests may appear

[2] We assume that the set of issues considered by the distributive policy is exogenously given. In any case, our model will allow to give zero weight (not considering) to anyone of them.

[3] We will also refer to a_i as the vector of characteristics of player i. By priorities of player i we will refer to the order induced by these characteristics.

[4] So, we analyze bargaining games where a specified quota is the consensus requirement for a proposal to succeed. See Baron and Ferejohn [1], Harrington [8] and Cardona-Coll and Mancera [3] for an analysis of multilateral bargaining games where simple majority is required.

when deciding the sizes of the public goods which are produced. We use this interpretation of the model throughout this paper.

Remark 1 Another interesting environment where our model applies is the one where some distributive policy is made on grounds of some proportional rule. This could be the case when some central government shares some budget to local governments based on their relative needs. Each agent has some exogenous characteristics measuring her relative needs on different issues. Thus, once there is agreement on the part of the budget dedicated to cover the needs of some specific issue, the players receive a surplus which is proportional to their characteristics. An example could be when the European Union decides how to share some fixed and divisible budget among its members: the decision considers the characteristics (which measure their needs) of the countries affected by the distributive policy; and proportionality to these characteristics, each country receives part of the surplus.[5] Of course, in this setting, the set of characteristic is restricted to vectors satisfying $\sum_{i=1}^{n} a_i(j) = 1$ for any $j \in K$, where $a_i(j)$ represents the relative claim on issue j that player i has.

The bargaining consists of deciding the priorities of the public policy: the players must choose the part of the budget which is dedicated to each issue. They bargain over the values of the vector of priorities $x = (x(1), \ldots, x(k)) \in \Delta^{k-1}$. When some x is agreed at some period t, then the utility that a particular agent i gets is $u_i(x;t)$. The players are risk neutral. Because they may fail to distribute the surplus in the first period, the time preference of the players may play a role. We assume that they have stationary preferences with a common discount factor $\delta \in [0,1)$. Accordingly, the utility of player i is given by $u_i(x;t) = \delta^{t-1}\left[\sum_{j=1}^{k} a_i(j) \cdot x(j)\right]$. Thus, rivalry appears in deciding the amount of each public good but not on their consumption.

Consider the case where $a_i(j) = a_i(l)$ for any $j, l \in K$, and for any $i \in I$. It is clear that in this case there is no possible bargaining. The players will receive a utility that is exactly equal to their exogenous parameters defining their preferences. Thus, there are situations where the bargaining has no sense, since the utility profile is deterministic. Another degenerate situation would appear when there exists some issue $j \in K$ satisfying $a_i(j) > a_i(l)$ for any $i \in I$ and for any $l \in K$, $l \neq j$. In this case, there is no conflict among the players, who unanimously agree on $x(j) = 1$. However, when $k \geq 2$, $a_i(j) > a_i(l)$ and $a_z(j) < a_z(l)$ for some $i, z \in I$, $j, l \in K$, the rules governing the bargaining can affect the resulting outcome inducing to different assignments of the budget. We assume that $a_i(j) \neq a_i(l)$ for all $i \in I$ and for all $j, l \in K$, $j \neq l$.[6]

The equilibrium concept we consider is the sophisticated equilibrium. That is, when the players simultaneously veto or accept some proposal, then they do not use weakly dominated strategies.[7] By

[5] The European "Compensation Funds" could be an example. There is also a fixed cutoff for a country to have the option to receive part of these funds. However, the underlying setup can be considered as identical. Also, the usual negotiations among EU countries on agricultural policy could be reinterpreted within our setting.

[6] Notice that the vectors of characteristics are exogenously given. If we consider that they are drawn from some smooth distribution function which has full support, then the set of vectors with some components being equal has zero measure.

[7] As in Moulin [10], we use the term of sophisticated equilibria to refer to the subgame perfect equilibrium strategies

modelling the voting stage as one of simultaneous moves with only the outcome being observed, we introduce some type of stationarity in the model since the players cannot condition their actions to the specific action of some player(s) at the voting stage. However, non stationary strategies are not excluded yet.

2.1 The Two-Characteristics Case

In this section, we consider the case where just two public projects are available. Hence, each agent $i \in I$ is characterized by a relevant pair $a_i = (a_i(1); a_i(2))$. There is a recognition rule which selects the proposer, each player with equal probability. This player makes a proposal $(x(1); x(2)) \in \Delta$ which indicates the part of the budget assigned to each (public) issue. Notice that once $x(i) \in [0; 1]$ is selected, by feasibility $x(j) = 1 - x(i)$ is also determined. For simplicity, denote by x the part of the budget assigned to the first issue.

Given a vector $a = (a_1; a_2; ...; a_n)$ of individual characteristics, define

$$r(1;2) = (r_1(1;2); r_2(1;2); ...; r_n(1;2))$$

as the vector such that [8]

$$r_i(1;2) = \frac{a_i(2)}{a_i(1) - a_i(2)} \text{ for any } i \in I. \tag{1}$$

Since the proposers are randomly chosen at each period, without loss of generality (w.l.o.g.) we can reorder the players such that

$$r_1(1;2) \geq r_2(1;2) \geq ...... \geq r_n(1;2). \tag{2}$$

Define $\Omega^+(1;2) = \{i \in I \mid r_i(1;2) \geq 0\}$ and $\Omega^-(1;2) = \{i \in I \mid r_i(1;2) < 0\}$. These two sets of players (groups) have the property that players in the same group have the same preference ordering over different outcomes (they have common interests). Denote by $\beta(1;2)$ the proportion of players in $\Omega^+(1;2)$. I.e., $\beta(1;2) = m/n$ where $m = |\Omega^+(1;2)| \in [1; n)$ denotes the cardinality of $\Omega^+(1;2)$. Hence, $\beta(1;2)$ is also the proportion of players preferring x over $\overline{x}$ whenever $x > \overline{x}$. Next Lemmas provide some properties of the preferences of the players that will facilitate the specification of the equilibria strategies. To simplify notation, we avoid the arguments of r_i and β through this section.

Lemma 1 If $w \in I$ with $w \leq m$ (strictly) prefers x at the current period to $\overline{x}$ at next period, then any $j \in I$ with $j \leq w$ also prefers it.

Proof. Player w (strictly) prefers x to $\overline{x}$ at next period, so the next inequality must hold

$$x a_w(1) + (1 - x) \cdot a_w(2) > \delta \cdot [\overline{x} \cdot a_w(1) + (1 - \overline{x}) \cdot a_w(2)] \tag{3}$$

that yield a sophisticated outcome in Farquharson's [4] sense. The sophisticated outcomes in simultaneous voting contexts have been shown to coincide with the outcomes that result in equilibrium when the voting stage is considered sequential (see Sloth [13]). However, in our context the actions in the voting stage are not observed, so that this equivalence does not hold. For an extensive description of sophisticated voting see Farquharson [4].

[8] The definition of this value and posterior reordering of the players according to it, is convenient for the exposition of the proofs.

Let's now consider some $j \in I$, $w \geq j$, and assume that she prefers $\overline{x}$ at next period to x at the current period.

$$x \cdot a_j(1) + (1 - x) \cdot a_j(2) \leq \delta \cdot [\overline{x} \cdot a_j(1) + (1 - \overline{x}) \cdot a_j(2)] \tag{4}$$

This implies that

$$x \leq \delta \cdot \overline{x} - r_j (1 - \delta) \tag{5}$$

Since $r_w \leq r_j$, this gives

$$x \leq \delta \cdot \overline{x} - r_w (1 - \delta) \tag{6}$$

contradicting inequality (3). ■

Lemma 2 *If $w \in I$ with $w > m$ (strictly) prefers x at the current period to $\overline{x}$ at next period, then any $j \in I$ with $j \geq w$ also prefers it.*

Proof. The proof is similar to the proof of the previous Lemma, just with the difference that now $a_i(1) - a_i(2) < 0$, and thus it is omitted. ■

Depending on the relationship between $m; n$ and q we have different scenarios. For instance, in case that $q \leq n - m$ and $q \leq m$ (therefore $q \leq n/2$) any player can propose her most preferred weight (either $x = 0$ or $x = 1$) and this proposal being accepted. Therefore, the expected equilibrium share on the first issue becomes $m/n = \beta$, being the expected utility of any player $i \in I$

$$\beta a_i(1) + (1 - \beta) a_i(2): \tag{7}$$

In this case, the players in each group do not need to concede to their "opponents" in order to reach an agreement. So, the usual trade-off of the bargaining is missed when the majority requirements are too weak. However, when stronger majority consensus are required for a proposals to succeed, then the players may need to concede to their opponents in order to reach an agreement.

To give some intuition about the equilibrium strategies, consider first that bargaining takes place over a finite set of T periods. When no agreement has been reached at period $t = T - 1$, then extreme priorities ($x \in \{0; 1\}$) will be proposed by the members of any group at next period. The probability that $x = 1$ is selected is then β. Furthermore, β coincides with the expected share (on characteristic 1) at $t = T$, yielding an expected utility of $\beta a_i(1) + (1 - \beta) a_i(2)$ for any player $i \in I$.

Denote by μ_t the vector of expected priority of issue 1 at period t (which is uniquely determined at $t = T$). At period $t - 1$, player i will propose (in case she is the proposer) x^i_{t-1} that maximizes her utility and satisfies[9]

$$x^i_{t-1} a_z(1) + \left(1 - x^i_{t-1}\right) a_z(2) \geq \delta [\mu_t a_z(1) + (1 - \mu_t) a_z(2)]; \; z \in I, \text{ for at least } q \text{ players.} \tag{8}$$

[9] Notice that, given our generic assumptions on the vectors of characteristics, this value is uniquely determined.

The Lemmas 1 and 2 indicate the cutoff for a proposal to be accepted by the players in any group. From them, one can easily show that a player $i \in \Omega^+(1;2)$ $(i \leq m)$ will choose

$$x^+_{t-1}(\theta_t) = \min\left\{\delta\theta_t - r^+(q)\cdot[1-\delta];1\right\}, \tag{9}$$

where

$$r^+(q) = \begin{cases} -1 & \text{if } q \leq m \\ r_{n+(m+1)-q} & \text{otherwise} \end{cases}$$

Similarly, a player $i \in \Omega^-(1;2)$ will choose

$$x^-_{t-1}(\theta_t) = \max\left\{\delta\theta_t - r^-(q)\cdot[1-\delta];0\right\} \tag{10}$$

where

$$r^-(q) = \begin{cases} +1 & \text{if } q \leq n-m \\ r_{q+m-n} & \text{otherwise} \end{cases}$$

Therefore, given (9) and (10)

$$\theta_{t-1} = \beta\cdot x^+_{t-1}(\theta_t) + (1-\beta)\, x^-_{t-1}(\theta_t) \tag{11}$$

Using the previous equation, we can recursively find (with $\theta_T = \beta$) a unique θ_1 as a function of $T; r^+(q); r^-(q)$ and δ in the finite horizon game.

Notice also that (for the infinite horizon game) any stationary equilibrium vector of expected weights θ_e must satisfy $\theta_e = \beta x^+_t(\theta_e) + (1-\beta)\, x^-_t(\theta_e)$. In fact, one can show that there exists a unique sophisticated equilibrium in the infinite horizon game which coincides with the stationary equilibrium.

Proposition 3 If just two public goods are available, then there exists (generically) a unique sophisticated equilibrium.

Proof. Let $\underline{\theta}$ and $\overline{\theta}$ denote the minimal and maximal expected values of x in any equilibrium. Since $\overline{\theta}$ is the highest expected share assigned to issue 1, (by using Lemma 1 and Lemma 2) no player in $\Omega^-(1;2)$ will accept any proposal greater than $x^+\left(\overline{\theta}\right)$, and $x^-\left(\overline{\theta}\right)$ will be accepted for sure by players in $\Omega^+(1;2)$.[10] This implies that

$$\overline{\theta} \leq \beta\cdot x^+\left(\overline{\theta}\right) + (1-\beta)\cdot x^-\left(\overline{\theta}\right). \tag{12}$$

Similarly,

$$\underline{\theta} \geq \beta\cdot x^+(\underline{\theta}) + (1-\beta)\cdot x^-(\underline{\theta}) \tag{13}$$

where $x^+(\theta) = \min\{\delta\theta - r^+(q)[1-\delta];1\}$ and $x^-(\theta) = \max\{\delta\theta - r^-(q)[1-\delta];0\}$.

[10] This is true if we consider that only the outcome of the voting stage is made public, and players use non (weakly) dominated strategies. As usual, notice that in equilibrium, an individual that is indifferent between continuation and the proposed share must accept the proposal.

Assume that (12) holds with strict inequality. This means that there is no subgame where $x^+\left(\bar{\theta}\right)$ is chosen with probability β and $x^-\left(\bar{\theta}\right)$ with probability $(1-\beta)$. Therefore, $\bar{\theta}$ is not the maximal expected value for x in any equilibrium, which is a contradiction. Similarly, (13) must hold with equality.

Define the function $h : [0,1] \to [0,1]$ as

$$h(\theta) = \beta \cdot x^+(\theta) + (1-\beta) \cdot x^-(\theta).$$

By the above argument, $\bar{\theta}$ and $\underline{\theta}$ must satisfy

$$h\left(\bar{\theta}\right) = \bar{\theta} \text{ and } h(\underline{\theta}) = \underline{\theta}. \tag{14}$$

The function h is continuous and increasing since it is a convex combination of two continuous and increasing functions, α^+ and α^-. Moreover, since $0 \leq \frac{\partial x^+(\theta)}{\partial \theta} < 1$ and $0 \leq \frac{\partial x^-(\theta)}{\partial \theta} < 1$, we have that $0 \leq \frac{\partial h(\theta)}{\partial \theta} < 1$. Also, $h(0) > 0$ and $h(1) < 0$. Therefore (by using a fixed point argument), there exists a unique $\theta^* \in [0,1]$ satisfying $h(\theta^*) = \theta^*$. Consequently, $\bar{\theta} = \underline{\theta} = \theta^*$. ∎

How would look like the equilibrium shares? Next example shows that the proportion of players in each group can determine the expected shares on each issue when there exists some player in each group having zero receiving zero utility from one issue.

Example 1 Consider the case where there exist $i, z \in I$ such that

$$a_i(1) > a_i(2) = 0 \text{ and } 0 = a_z(1) < a_z(2).$$

and unanimity is required for a proposal to succeed.

In equilibrium, we know that

$$\theta^* = \beta\left[\min\{\delta\theta^* + (1-\delta), 1\}\right] + (1-\beta)\left[\max\{\delta\theta^*, 0\}\right],$$

and therefore,

$$\theta^* = \beta\left[\delta\theta^* + (1-\delta)\right] + (1-\beta)\cdot\delta\theta^*,$$

which yields

$$\theta^* = \beta.$$

That is, the initial proportion of players in each group determines the expected shares.

Even if the proportion of players in each group has an impact on the equilibrium expected weights, the values of r_i $(i \in I)$ are also important. Next example shows that θ^* depends on these values, and not only on the proportion of players in each group.

Example 2 Consider a unanimity bargaining game with $r_m > 0$ and $r_{m+1} = -1$. In this case,

$$\theta^* = \beta\left[\delta\theta^* + (1-\delta)\right] + (1-\beta)\left[\max\{\delta\theta^* - r_m(1-\delta), 0\}\right];$$

and therefore,

$$\theta^* \le \beta - r_m(1-\delta) < \beta.$$

That is, the fact that $a_{m+1}(1) = 0$ and $a_m(2) > 0$ produces such vector of expected weights.

Remark 2 One can easily show that the equilibrium expected share on issue 1, θ^*, is (weakly) decreasing in r_j, for $j \le m$ and (weakly) increasing in β and r_k, where $k \ge m+1$. Therefore, any player in $\Omega^+(1,2)$ would benefit from the existence of another player i in the same group having $a_i(2) = 0$.

2.2 More than Two Characteristics

The important property of the two-issue bargaining game is that for any three alternatives majorities are stable (in the sense that the players in the same group have the same preference ordering over different outcomes). When there are more than two issues this "stability" rarely appears. Given some vectors of three characteristics, majorities can be unstable when we consider the preferences of the players over the triplets $(1,0,0), (0,1,0)$ and $(0,0,1)$. Moreover, two players preferring $(1,0,0)$ can have different preferences over convex combinations of the three extreme alternatives. As a consequence, it is not possible to separate the players into two groups with common interests. This was not the case of the previous section where the majority relation was "stable".

Of course, when the number of players involved in the bargaining are just two, again the preferences of the players are polarized into two groups. Not surprisingly, a unique equilibrium outcome appears in such a context, independently of the number issues (see Proposition 6 in Appendix A). We observe that the minimal value of the agents' characteristics (the minimal marginal utility) plays the same role as an external option in the standard alternating offers bargaining game, when it can be exercised by the player who is the responder (see Binmore [2]).[11] However, for more than two players, the existence of majority cycles over different assignments of the budget induces to multiple equilibria for players patient enough, as it is shown in the next (generic) example.

Example 3 Consider a unanimity bargaining game and let $a_1 = (.7, .4, .2)$, $a_2 = (.1, .5, .3)$ and $a_3 = (.2, .1, .5)$. The extreme payoffs correspond to the vector of weights $x_1 = (1, 0, 0)$, $x_2 = (0, 1, 0)$ and $x_3 = (0, 0, 1)$, which assign utilities $b = (.7, .1, .2)$, $c = (.4, .5, .1)$ and $d = (.2, .3, .5)$ respectively. We now show that any interior point in the convex hull of these vectors can be supported in equilibrium for some δ close enough to 1. Let $e = (e_1, e_2, e_3)$ be a payoff configuration in the interior of the convex hull of b, c

[11] By accepting the worst vector of weights, a player can guarantee this amount when $n = 2$. So, it is clear that in such case it could be considered as an outside option. However, this relationship is not clear when there are more than two players because of the unanimity requirement. Notice also that the uniqueness appears when the player who can opt to her external option is not the proposer. Otherwise multiple equilibria are sustained (see Ponsatí and Sákovics [11]).

and d, and consider $y=(y_1;y_2;y_3)$ and $z=(z_1;z_2;z_3)$ also in this convex hull such that $e_1>y_1=z_1$, $e_2=y_2<z_2$ and $e_3=z_3<y_3$, with $\sum_{i=1}^{3} w_i=1$, where w_i denotes the $i-$th component of the vector w, for $w=e;y;z$ and $i=1;2;3$. Consider the strategies "play e" given by:[12]

(i) Any player proposes e.

(ii) When e is proposed the players accept it.

(iii) If either 2 or 3 propose w satisfying $w_1<\delta e_1$ then e is played again whenever it is refused.

(iv) If 2 proposes w satisfying $w_1\geq\delta e_1$ and $w_3<\delta y_3$ then y is played whenever it is refused (with similar strategies to (i) to (v)).

(v) If 3 proposes w satisfying $w_1\geq\delta e_1$ and $w_2<\delta z_2$ then z is played if it is refused (with similar strategies to (i) to (vi)).

(vi) If 1 proposes w satisfying $w_2<\delta z_2$ then z is played and if $w_3<\delta y_3$ then y is played if it is refused (with similar strategies to (i) to (vi)).

Notice that when δ is close to 1 then neither $w_1\geq\delta e_1$, $w_3\geq\delta y_3$ and $w_2\geq e_2$ nor $w_1\geq\delta e_1$, $w_2\geq\delta z_2$ and $w_3\geq y_3$ nor $w_2\geq\delta z_2$, $w_3\geq\delta y_3$ and $w_1\geq e_1$can be simultaneously satisfied. Hence, the previous strategies constitute a sophisticated equilibrium.

3 Issue-by-Issue Bargaining

Consider n players characterized by the vectors $a_i=(a_i(1);a_i(2);:::;a_i(k))$ for any $i\in I$, bargaining on how to allocate a budget of 1 among k issues. Again, they have to choose some $x=(x(1);x(2);::;x(k))\in\Delta^{k-1}$. However, the rules governing the bargaining are quite different. Now, these shares assigned to each issue are chosen sequentially and independently (except by the feasibility restriction).[13] We call the $j-$th bargaining round to the set of periods where the players bargain over the share (or priority) assigned to issue $j\in K$.

At first period the first round starts, with players bargaining over $x(1)$. A proposer $i\in I$ is randomly chosen, with equal probability each player. She makes proposal $x_i(1)$, and then the rest of players simultaneously either accept or reject it. If at least q players accept it then a second round starts at the same period with the players bargaining over $x(2)$. Otherwise, the game moves to the next period, where a new proposer is randomly chosen. At the beginning of the $j-$th round there is a vector of accepted shares $(x^*(1);:::;x^*(j-1))$ and a player $z\in I$ is chosen as the proposer, with equal probability each. She makes a proposal $x_z(j)$ which can be accepted or rejected by the players, and so on. The shares that the players may propose in each round j must satisfy feasibility, i.e. $x(j)+\sum_{l=1}^{j-1}x^*(l)\leq 1$. Denote by $\mu_j=1-\sum_{l=1}^{j-1}x^*(l)$ the percentage of the budget which remains not assigned at the beginning of the

[12] The following set of actions does not completely define a strategy profile. However, it is enough for our purposes.
[13] This idea of partial agreements is also developed by Winter [14].

$j-$th round. The bargaining game ends once $x^*(k-1)$ is chosen, being $x^*(k) = \rho_k$, or when $\rho_j = 0$ for some $j \in \{1; k-1\}$.

We consider that discount occurs just when some proposal is rejected.[14]

Notice that using a backwards argument, we deal we a sequence of two-issue bargaining games: the players must agree on the share assigned to one issue and the share left to continuation, which must be assigned among other issues. So, one might think that this would restore uniqueness. However, this strongly depends on the exact meaning of an accepted proposal.

By modelling the bargaining through a sequence of partial agreements, how these agreements are implemented has a crucial effect on the equilibrium outcomes. A first possibility is that, even if some partial agreements have been reached, public projects are not implemented until all the budget is assigned. In such a case, previous agreements can affect the equilibrium shares of future issues because of the impatience of the players in getting part of their (conditionally assigned) utility. A second possibility is when partial agreements are implemented immediately, or in other words being non contingent to a complete agreement. These two contents of partial agreement have very different implications: while in the first case, past agreements have an impact on future rounds of bargaining, in the second case they do not.

3.1 Non-Contingent Partial Agreements

We first analyze the case of partial agreements not being contingent to the end of negotiations. In this situation, once they decide to assign part of the budget to some issue then they immediately get some utility from the implementation of such public project. Next, they will bargain over the part of the budget that remains not assigned. Thus, the "new" bargaining game has exactly the same properties as the original bargaining game with the only difference that the budget is (eventually) smaller and that the heterogeneity of the players decreased one dimension. Except for these features, what happened in previous rounds does not affect the current bargaining game. Therefore, the utility of a player is now given by $u_i(x^*; t) = \sum_{j=1}^{k} \delta^{t_j} x^*(j) \cdot a_i(j)$, where t_j is the period in which the players agree on $x^*(j)$.

Assume that the players have agreed on $(x^*(1); \ldots; x^*(k-2))$; and consider the subgame starting at the $(k-1)-$th round. In this subgame, the equilibrium expected shares are given by $\rho_{k-1} \cdot (\theta^*(k-1;k); 1-\theta^*(k-1;k))$, where $\theta^*(k-1;k)$ denotes the expected share on primitive $(k-1)$ given by Proposition 3, which results in equilibrium when the players bargain just over $x(k-1)$ and $x(k)$.[15] Consider now the $(k-2)-$th bargaining round. We can redefine the (sub)game as an equivalent bar-

[14] If we consider the discount factor as a measure of the impatience of the players, then just non accepted proposals can affect their subjective payoffs. Notice that there must be different rounds at the same period (and obviously the same round can take several periods).

[15] Notice that the normalization to 1 in Proposition 3 has no qualitative effects on the outcome.

gaining game with two issues, $a(k-2)$ and $b(k-1)$, where $b(k-1)$ satisfies[16]

$$b_i(k-1) = \theta^*(k-1;k)\cdot a_i(k-1) + [1-\theta^*(k-1;k)]\cdot a_i(k) \text{ for any } i \in I. \tag{15}$$

Hence, at the $(k-2)-$th round the players face to a bargaining problem over $\mu_{k-2}\cdot x(k-2)$; which uniquely determines the expected utility at the next bargaining round, which is equal to $\mu_{k-2}[1-x(k-2)]\cdot b_i(k-1)$ for any $i \in I$. Since the players receive utility from any issue when the share assigned to it is approved, they cannot affect $\theta^*(k-1;k)$ when selecting $x(k-2)$. So, the vector of expected shares coincides with the one derived in Proposition 3, uniquely determined whenever $b_i(k-1) \neq a_i(k-2)$ for any $i \in I$, which is generically assumed.

Define recursively

$$b_i(j) = \theta^*(j;j+1)\cdot a_i(j) + [1-\theta^*(j;j+1)]\cdot b_i(j+1) \text{ for any } i \in I. \tag{16}$$

where $(\theta^*(j;j+1); 1-\theta^*(j;j+1))$ is the unique vector of equilibrium expected priorities of the bargaining game when just two characteristics of the players, $a_i(j)$ and $b_i(j+1)$, are considered. At each bargaining round j, the preferences of the players are polarized over $x(j) = \mu_j$ and/or $x(j) = 0$ (which equals to assign μ_j to continuation). Consequently, Proposition 3 can be inductively applied yielding next result.

Proposition 4 *The bargaining game with sequential non contingent partial agreements has generically a unique sophisticated equilibrium.*

This uniqueness result uses the fact that the order in which the players bargain over the priorities of the public policy (i.e. the agenda of the bargaining) is fixed. However, the agenda has a strong impact on the equilibrium allocation, as is shown in the next example.[17]

Example 4 Consider a unanimity bargaining game with $a_1 = (4;2;1)$, $a_2 = (4;5;3)$ and $a_3 = (2;3;6)$. Assume that the players bargain first on $x(1)$. Direct computations show that in equilibrium $x^*(1) = 82/117$, $x^*(2) = (35/117)(5/6)$ and $x^*(2) = (35/117)(1/6)$. If the players must choose first $x(2)$, then we have that in equilibrium $x^*(1)$ tends to 1 as δ goes to 1. And finally, if the players first bargain over $x(3)$, the shares converge to $x^*(1) = (19/20)(1/3)$, $x^*(2) = (19/20)(2/3)$ and $x^*(1) = 1/20$ as players become patient.

3.2 Contingent Partial Agreements

The case where partial agreements are contingent to a complete agreement enters a new feature: the impatience of the players introduces a relationship between past decisions and future equilibria outcomes.

[16] By defining this "new" characteristic, we observe the exact relationship between the marginal utility of the issue considered at this round and the marginal utility of continuation.

[17] The effects of the agenda on the equilibrium outcomes have been extensively analyzed in voting contexts (see Moulin [10]). Ferejohn et al. [5] analyze a special legislative environment where such agenda has no effects on the final outcome. See also Fershtman [6].

Now, the bargaining over the share of some specific issue is not only affected by future assignments of the budget, but also depends on how some priorities have been assigned to other issues. A player who obtained a high contingent utility through previous partial agreements behaves as if she was more impatient than a player who obtained a smaller contingent utility. Consequently, the inductive argument of the previous section, which bears on the fact that future agreements do not depend on actual decisions, cannot be used to analyze the problem.[18]

Since the players cannot consume their agreed upon shares until the agreement is global, we have that (as in section 2) the utility of the players is given by $u_i = \sum_{j=1}^{k} \delta^t x^*(j) \cdot a_i(j)$ where t indicates the period in which a complete agreement is reached, and $x^* = (x^*(1); ...; x^*(k))$ denotes the agreed vector of priorities.

Consider the $(k-1)-$th round. That is, $(k-2)$ priorities have been chosen and the players face to a decision over the shares of just two issues. There is still (w.l.o.g.) a majority players, say m, who prefer to assign all the budget to issue $(k-1)$. However, the unique sophisticated outcome of the $(k-1)-$th bargaining round does not coincide with the one given by Proposition 3. Now, the shares assigned to previous issues will affect the bargaining at the current period.

Define by $s = (s_1; s_2; ...; s_n)$ the vector satisfying

$$s_i = \frac{g_i + a_i(k)}{a_i(k-1) - a_i(k)} \text{ for any } i \in I, \tag{17}$$

where

$$g_i = \left(\sum_{j=1}^{k-2} \alpha^*(j) a_i(j) \right) / \rho_{k-1}. \tag{18}$$

As in section 2.1., we can reorder (w.l.o.g.) the players such that

$$s_1 \geq s_2 \geq .... \geq s_n \tag{19}$$

Using a similar reasoning as in section 3.1, one can show that given $x^*(1); ...; x^*(k-2)$, there exist a unique equilibrium vector of expected weights $\rho_{k-1}(\theta_c^*; 1-\theta_c^*)$ defined by[19]

$$\theta_c^* = (m/n) \cdot x^+(\theta_c^*) + [1 - (m/n)] \cdot x^-(\theta_c^*) \tag{20}$$

where $x^+(\theta_c^*) = \min\{\delta\theta_c^* - s^+(1-\delta); 1\}$ and $x^-(\theta_c^*) = \max\{\delta\theta_c^* - s^-(1-\delta); 0\}$, with

$$s^+(q) = \begin{cases} -1 & \text{if } q \leq m \\ s_{n+(m+1)-q} & \text{otherwise} \end{cases}$$

[18] Notice also that the genericity assumptions we make to avoid indifferences could not necessarily be satisfied because of the endogeneity of the actions.

[19] Now, equation (3) and similars in the proof of Lemma 1 become

$$g_w + \alpha a_w(1) + (1-\alpha) \cdot a_w(2) \geq \delta \cdot [g_w + \bar{\alpha} \cdot a_w(1) + (1-\bar{\alpha}) \cdot a_w(2)]$$

and

$$s^i(q) = \begin{cases} +\infty & \text{if } q \le n - m \\ s_{q+m-n} & \text{otherwise} \end{cases};$$

and where m denotes the number of players with $s_i \geq 0$.[20]

One can observe the exact implications of the two contents of partial agreements that we considered: while non-contingent partial agreements do not affect the negotiations on future rounds, the expected equilibrium shares depend (through g_i) on how different shares have been assigned in the past when partial agreements are contingent (now transfers among issues are possible).

Consider now the $(k-2)-$th bargaining round. We know that at the $(k-1)-$th bargaining round, the expected shares are $\mu_{k-1} \cdot (\theta_c^*(k-1;k); 1-\theta_c^*(k-1;k))$ where $\theta_c^*(k-1;k)$ is given by (20). We can redefine the game as a two-issue bargaining game, $a(k-2)$ and $b(k-1)$, where $b(k-1)$ satisfies

$$b_i(k-1) = \theta_c^*(k-1;k) \cdot a_i(k-1) + [1-\theta_c^*(k-1;k)] \cdot a_i(k) \text{ for any } i \in I. \tag{21}$$

Notice that $b_i(k-1)$ is a function of $x^*(k-2)$ because θ_c^* is. Given $x^*(1), \ldots, x^*(k-3)$, for every $x(k-2)$ we have a (not necessarily) different vector of redefined marginal utilities. Moreover, depending on $a = (a_1, \ldots, a_n)$ the value of $\theta_c^*(k-1;k)$ can either increase or decrease in $x^*(k-2)$. Therefore, the players cannot necessarily be polarized into two groups according to their preferences over $x(k-2)$. As a consequence, multiple equilibrium outcomes may appear in equilibrium.

Proposition 5 *The bargaining game with sequential contingent partial agreements has not generically a unique sophisticated equilibrium.*

Proof. See Appendix B. ■

As in the non contingent case, we still can reduce the utilities of the players to one dimension space when we consider three primitives (using a backwards argument). However, these preferences do not necessarily generate two unique stable groups of players so that reputation effects can appear in the model, leading to multiplicity. Different expected sophisticated outcomes are sustained because the players can condition their strategies to past actions. By considering secret voting we still allow for the strategies be dependent on who make a proposal. This fact is irrelevant if we have two stable subsets of players since the situation is somehow similar to the one where just two players bargain with different probabilities of being the proposer. However, in this new environment we can have majority cycles over the priorities assigned to the first characteristic (the preferences of the players are not necessarily single-peak with peaks at zero or one) and this allows to sustain multiple sophisticated equilibria.

4 Efficient Outcomes and Uniqueness

In the previous sections, we have shown that when partial agreements are immediately implemented, then uniqueness is generically achieved. On the other hand, when a complete agreement is required for the

[20] Notice that m is not affecte by the g_i's.

implementation of the public projects, then multiple sophisticated outcomes are sustained in equilibrium. Moreover, multiplicity is a consequence of the players using non-stationary equilibrium strategies. In this setup, it is not difficult to see that inefficiency is associated to this multiplicity. The inefficiency may appear because of delays in reaching an agreement. On the other hand, when agreements are non contingent, uniqueness implies that there is no delay. Thus, one could thing that this would imply efficiency. However, another type of inefficiency may appear because the players may assign inefficiently the shares among different issues. The reason is that utility transfers among different issues are not possible.

To see how this inefficiency may appear, we are going to consider a three-issue bargaining game with two groups of quite homogeneous players. Consider the situation where the players in group 1 have a vector of characteristics satisfying

$$a_1(1) > a_1(2) > a_1(3),$$

and any player in group 2 is characterized by

$$a_2(1) < a_2(2) < a_2(3).$$

In this framework, we can have two different situations:

1. $(a_1(2), a_2(2)) > \lambda (a_1(1), a_2(1)) + (1 - \lambda)(a_1(1), a_2(1))$ for some $\lambda \in [0,1]$.
2. $(a_1(2), a_2(2)) < \lambda (a_1(1), a_2(1)) + (1 - \lambda)(a_1(1), a_2(1))$ for some $\lambda \in [0,1]$.

In the first case, an efficient outcome must assign a share of zero, either to the first characteristic or to the third one, while in the second case, an efficient outcome must be such that $x_2 = 0$. The question is that if there is any agenda that yields such outcome.

It is not difficult to see that, in both cases, a necessary condition to get an efficient SSPE outcome is that the players negotiate first on the second issue. Additional conditions in the first case are,

$$\begin{aligned} \mu_{13} \cdot a_1(1) + (1 - \mu_{13}) \cdot a_1(3) &\geq a_1(2) \\ \mu_{13} \cdot a_2(1) + (1 - \mu_{13}) \cdot a_2(3) &\geq a_2(2) \end{aligned}$$

where μ_{13} denotes the expected priority assigned to issue 1 when the players bargain over the share assigned to 1 and assigning the remaining to issue 3. In such a case, the share assigned to issue 2 will be zero.

In the second case, a sufficient condition for the SSPE outcome being efficiency is that issue 2 is first in he agenda and

$$\begin{aligned} \mu_{13} \cdot a_1(1) + (1 - \mu_{13}) \cdot a_1(3) &\leq a_1(2) \\ \mu_{13} \cdot a_2(1) + (1 - \mu_{13}) \cdot a_2(3) &\leq a_2(2) \end{aligned}$$

Now, all the budget is assigned to issue 2.

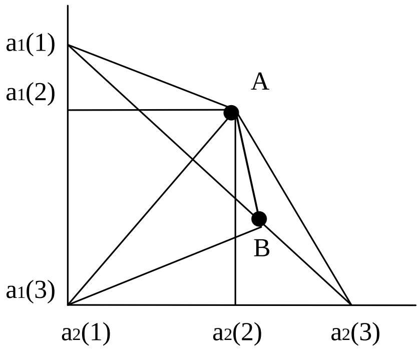

Figure 1:

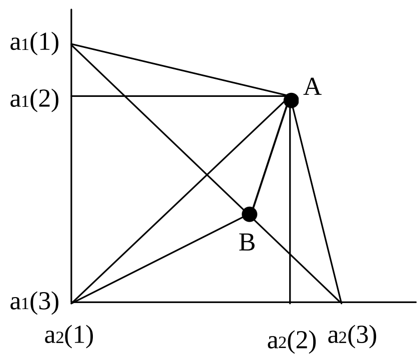

Figure 2:

Figures 1 and 2 illustrate the conditions in the …rst case. Assuming that the players negotiate …rst on issue 2; we know that μ_{13} is independent of x_2. The point A is the vector of utilities where $x_2 = 1$, while B is the one where $x_2 = 0$. Any intermediate point is the pair of utilities when x_2 2 $(0; 1)$. Thus the su¢cient condition requires that all these points being smaller than $(a_1(2); a_2(2))$. This is the case in Figure 1. Thus, assigning a share smaller than 1 to x_2 is strictly dominated for the two players. However, in Figure 2 there is one player (player 1) who prefers to assign 0 to issue 2 while the other prefers $x_2 = 1$, implying that an ine¢cient outcome (some point between A and B) is reached.

We observe that e¢ciency may be guaranteed in some (very demanding) cases, which share the particularity that the issue which is not preferred for any player is negotiated …rst. In all other agendas, the bargaining game yields an allocation which gives some positive expected share to all issues, implying ine¢ciency. Notice that the second issue is not the worst issue for any player. Thus, even if we cannot make a concluding statement, intuition suggests that to get e¢ciency we must put …rst in the agenda the "less con‡icting" issues. A similar conclusion has been pointed out by Winter in a di¤erent setup, in the sense that the more important issues must be put …rst on the agenda. Of course, the existence of such an issue in our setting will make all our analysis trivial. However, it seems that in a more con‡icting framework the idea of …rst bargaining on less con‡icting issues may be common in both models.

A very special case where e¢ciency may be guaranteed is the distributive policy framework. As we notice in section 2, a special case of our model is the one were a set of players must decide how to share a …xed and divisible amount among themselves according to proportional rules. The game is as follows: the budget can be assigned to di¤erent issues. Each player has some relative claim on each issue which is proportional to some objective variable (number of schools in her district, population, etc...): once part of the budget is thus assigned to an speci…c issue, then players receive a proportion (indicating the relative claim) of this part of the budget.

In this setup, we have that the set of relative claims on each issue add up 1. That is, $\sum_{i=1}^{n} a_i(j) = 1$ for any j 2 K. This implies that for any vector of priorities, the utility pro…le belongs to the frontier of the set of feasible utility pro…les. Thus, when an agreement is achieved without delay, e¢ciency is guaranteed. This is the case when partial agreements are not contingent to a complete agreement. Otherwise, delays can occur in some non-stationary equilibria.

There are other situations were uniqueness guarantees e¢ciency. It is the case where the preference pro…les are such that all feasible orderings are present. That is, any feasible ordering over extreme shares of the budget (i.e., all the budget assigned to just one issue) is represented by the preferences of at least one player. In such cases, any vector of shares must be e¢cient, since by rearranging this vector of shares there must be always a player whose utility decreases. Thus, we cannot have ine¢ciency because of a bad assignment of the priorities.

5 Final Remarks

We have modeled the situation where a set of players bargains over the priorities that a public policy assigns to di¤erent issues. We have shown that if for an agreement to be implemented a complete agreement is required, then (generically) multiple sophisticated equilibria appear. However, uniqueness is generically achieved when the bargaining is ruled by a sequence of partial agreements which are immediately applied. The results bear on the possibility (or not) of polarizing the preferences of the players into two groups, so that for any three alternatives and any three players there are no majority cycles. Roughly speaking, uniqueness appears because two stable sets of players over di¤erent alternatives can be formed in any subgame.

In the framework we considered, uniqueness is a necessary condition for e¢ciency. Thus, in contrast with two-person multi-issue bargaining games, we …nd that issue-by-issue bargaining (and not global bargaining) is the bargaining rule that can achieve e¢ciency. In any case, neither issue-by-issue nor global bargaining guarantees e¢ciency.

When a unique outcome is attained in equilibrium, the outcome has been shown to be dependent on the order in which the priorities are chosen. Thus, the agenda has a crucial importance in determining the …nal allocation. An obvious extension of our model would consider how this agenda is chosen. Another issue that remains unexplored in the paper is the (possible inclusion) relationship between the equilibrium allocations in the contingent and the non contingent partial agreements cases. Notice that we have explored a particular bargaining model and show that multilateral bargaining through partial agreements can help to reduce the set of equilibrium outcomes. However, this does not say anything about the “fairness” of the …nal allocations.

Appendix A

Let $a_1 = (a_1(1), \ldots, a_1(k))$ and $a_2 = (a_2(1), \ldots, a_2(k))$. Denote by $\overline{a}_i = \max_j a_i(j)$ and $\underline{a}_i = \min_j a_i(j)$ for any $i = 1, 2$. Therefore, $u_i \geq \underline{a}_i$ for any $i = 1, 2$.

Proposition 6 If $n = 2$ and $k > 2$ then there exists an (sophisticated) equilibrium where only two characteristics have positive weight. Moreover, in any equilibrium vector of expected payoffs is the same.

Proof. Let denote by $\overline{e}_i$ and $\underline{e}_i$ the maximum and minimum expected payoff of player i in any equilibrium. In equilibrium, the next inequalities must also hold.

$$\overline{e}_i \leq (1/2)\left[\min\left\{1 - \delta\underline{e}_j, \overline{a}_i\right\} + \min\{\delta\overline{e}_i, \overline{a}_i\}\right] \tag{22}$$

and

$$\underline{e}_i \geq (1/2)\left[\max\{1 - \delta\overline{e}_j, \underline{a}_i\} + \max\{\delta\underline{e}_i, \underline{a}_i\}\right] \tag{23}$$

These inequalities yield $\overline{e}_i = \underline{e}_i = 1/2$ whenever either $\underline{a}_1 = \underline{a}_2$ or $\underline{a}_i \leq \delta/2$ for any i. Consider now, w.l.o.g., that $\underline{a}_1 < \underline{a}_2$ with $\underline{a}_2 > \delta/2$. In such case, $\overline{e}_1 = \underline{e}_1 = (1 - \underline{a}_2)/(2 - \delta) < 1/2$ whenever $\delta\underline{e}_1 \geq \underline{a}_1$ and $\overline{e}_1 = \underline{e}_1 = (1 - \underline{a}_2 + \underline{a}_1)/2 < 1/2$ otherwise. In any case, the expected payoff is uniquely determined. Moreover, the players can choose weights only on the characteristics where the players have a smaller and higher proportion such that

$$\alpha_2\overline{a}_1 + (1 - x_2)\underline{a}_1 = \delta e_1 \tag{24}$$

and

$$\alpha_1\underline{a}_2 + (1 - x_1)\overline{a}_2 = \delta e_2 \tag{25}$$

where $x_1 = x^+(\theta) = \min\{\delta\theta - r_2(1 - \delta), 1\}$ and $x_2 = x^-(\theta) = \max\{\delta\theta - r_1(1 - \delta), 0\}$ with r_i being calculated considering the proportions $\overline{a}_i$ and $\underline{a}_i$, and $\theta = (x_1 + x_2)/2$. ■

Appendix B

Proof of the Proposition 2.

We prove the multiplicity of equilibrium outcomes by constructing an example which can be extended to an open set of initial characteristics. Consider the following vectors of characteristics: $a_1 = (0, 0.4, 0.6)$, $a_2 = (0.3, 0.2, 0.4)$ and $a_3 = (0.7, 0.4, 0)$.

Once the players agree on the weight assigned to characteristic 1, we get

$$g_1(x) = 0, g_2(x) = \frac{3x}{(1 - x)} \text{ and } g_3(x) = \frac{7x}{(1 - x)}$$

which yield

$$s_1(x) = \frac{7x}{4(1 - x)}, s_2(x) = -\frac{4 - x}{2(1 - x)} \text{ and } s_3(x) = -3.$$

Thus, the expected value of x_2 will be

$$\mu(x) = \begin{cases} \frac{1}{3}\min\left\{\delta\mu + \frac{4-x}{2(1-x)}(1-\delta);1\right\} + \frac{2}{3}\max\left\{\delta\mu - \frac{7x}{4(1-x)}(1-\delta);0\right\} & \text{if } x \leq 2/5 \\ \frac{1}{3}\min\{\delta\mu + 3(1-\delta);1\} + \frac{2}{3}\max\left\{\delta\mu - \frac{7x}{4(1-x)}(1-\delta);0\right\} & \text{if } x \geq 2/5 \end{cases}$$

This will yield, for sufficiently players

$$\mu(x) = \begin{cases} \frac{2-4x}{3(1-x)} & \text{if } x \in [0;2/5] \\ \frac{6-13x}{6(1-x)} & \text{if } x \in [2/5;6/13] \\ 0 & \text{otherwise} \end{cases}$$

>From this, we can rewrite the expected payoffs of the players as a function of x. We get [21]

$$u_1(x) = \begin{cases} 0.466 - 0.33x & \text{if } x \in [0;2/5] \\ 0.4 - 0.166x & \text{if } x \in [2/5;6/13] \\ 0.6 - 0.6x & \text{if } x \in [6/13;1] \end{cases}$$

$$u_2(x) = \begin{cases} 0.266 + 0.166x & \text{if } x \in [0;2/5] \\ 0.2 + 0.33x & \text{if } x \in [2/5;6/13] \\ 0.4 - 0.1x & \text{if } x \in [6/13;1] \end{cases}$$

and

$$u_3(x) = \begin{cases} 0.266 + 0.166x & \text{if } x \in [0;2/5] \\ 0.4 - 0.166x & \text{if } x \in [2/5;6/13] \\ 0.7x & \text{if } x \in [6/13;1] \end{cases}$$

We observe that for any $x \in (0.2;1)$ there exist $\hat{x} \neq x$ such that $u_2(\hat{x}) = u_2(x)$. We want to show that there exist some values of the x that can be achieved as the expected equilibrium weight on the first characteristic.

Take $\overline{x}$ and $\overline{\overline{x}}$ in the interval $(0.2;1)$ such that $u_2(\overline{x}) = u_2\left(\overline{\overline{x}}\right)$. We want to see if these weights can be sustained in equilibrium. I.e.,

$$\overline{x} = \frac{1}{3}\left[x_1(\overline{x}) + x_2(\overline{x}) + x_3(\overline{x})\right]$$

and

$$\overline{\overline{x}} = \frac{1}{3}\left[x_1\left(\overline{\overline{x}}\right) + x_2\left(\overline{\overline{x}}\right) + x_3\left(\overline{\overline{x}}\right)\right]$$

where $x_i(x)$ denotes the equilibrium proposal of player i $(i = 1;2;3)$ when the expected equilibrium weight is $x \in \left\{\overline{x};\overline{\overline{x}}\right\}$.

[21] Now, subindexes correspond with the initial ordering of the players.

The equilibrium strategies which sustain such equilibria are such that the equilibrium expected outcome will change from $\bar{x}$ to $\bar{\bar{x}}$ when deviations are observed. These expected payoffs are sustained by strategies which are partially defined below.

Play $\bar{x}$:

(i) Player 1 proposes $x_1(\bar{x}) = \delta\bar{x} - (0.26/0.16)(1-\delta)$.

(ii) Player 3 proposes $x_3(\bar{x}) = \delta\bar{x} + (0.46/0.3)(1-\delta)$.

(iii) Player 2 proposes $x_2(\bar{x}) \in [x_1(\bar{x}); x_3(\bar{x})]$ such that $x_2(\bar{x}) = \bar{x} - x_1(\bar{x}) - x_3(\bar{x})$.

(iv) If either player 1 or 2 deviate and the proposal id refused then we move to "Play $\bar{\bar{x}}$". If 3 deviates and her proposal is refused then $\bar{x}$ is played again.

Play $\bar{\bar{x}}$:

(i) Player 1 proposes $x_1\left(\bar{\bar{x}}\right) = \delta\bar{\bar{x}}$.

(ii) Player 3 proposes $x_3\left(\bar{\bar{x}}\right) = \delta\bar{\bar{x}} + (0.2/0.3)(1-\delta)$.

(iii) Player 2 proposes $x_2\left(\bar{\bar{x}}\right) \in \left[x_1\left(\bar{\bar{x}}\right); x_3\left(\bar{\bar{x}}\right)\right]$ such that $x_2\left(\bar{\bar{x}}\right) = \bar{\bar{x}} - x_1\left(\bar{\bar{x}}\right) - x_3\left(\bar{\bar{x}}\right)$.

(iv) If either player 1 or 2 deviate and the proposal is rejected then we move to "Play $\bar{x}$". If 3 deviates and her proposal is rejected then $\bar{\bar{x}}$ is played again.

One can show, that for players patient enough, there exist values of $\bar{\bar{x}}\left(\bar{\bar{x}} > 0.6\right)$ such that these strategies constitute a sophisticated equilibrium. Moreover, the example we propose is generic since we can construct such equilibria in an open set of characteristics.

References

[1] Baron, D.P. and Ferejohn, J.A. (1989), Bargaining in Legislatures, American Political Science Review 83, 1181-1206.

[2] Binmore, K.G. (1987), Perfect Equilibria in Bargaining Models, in The Economics of Bargaining, edited by Binmore and Dasgupta, Oxford: Blackwell.

[3] Cardona-Coll, D. and Mancera, F.M. (2000), Demand Bargaining in Legislatures, Economic Theory 16, 163-180.

[4] Farquharson, R.(1969), Theory of Voting. New Haven: Yale University Press.

[5] Ferejohn, J., Fiorina, M. and McKelvey, R. (1987), Sophisticated Voting and Agenda Independence in the Distributive Politics Setting, American Journal of Political Science 31, 169-173.

[6] Fershtman, C. (1990), The Importance of the Agenda in Bargaining, Games and Economic Behavior 2, 224-238.

[7] Kreps, D. and Wilson, R. (1982), Reputation and Imperfect Information, Journal of Economic Theory 27, 253-279.

[8] Harrington Jr., J.E. (1990), The Power of the Proposal Maker in a Model of Endogenous Agenda Formation, Public Choice 64, 1-20.

[9] Inderst, R. (2000), Multi-issue Bargaining with Endogenous Agenda, Games and Economic Behavior 30, 64-82.

[10] Moulin, H. (1986), Choosing from a Tournament, Social Choice and Welfare 3, 271-291.

[11] Ponsatí, C. and Sákovics, J. (1998), Rubinstein Bargaining with Two-Side Outside Options, Economic Theory 11, 667-672.

[12] Rubinstein, A. (1982), Perfect Equilibrium in a Bargaining Model, Econometrica 50, 97-109.

[13] Sloth, B. (1993), The Theory of Voting and Equilibria in Noncooperative Games, Games and Economic Behavior 5, 152-69.

[14] Winter, E. (1997), Negotiations in Multi-Issue Committees, Journal of Public Economics 65, 323-342.